A Generalized Maxwell-Slip Friction Model appropriate for Control Purposes

Vincent Lampaert, Farid Al-Bender, Jan Swevers
Mechanical Engineering Department, Katholieke Universiteit Leuven
Celestijnenlaan 300B, B3001 Heverlee, Belgium
e-mail: Vincent.Lampaert@mech.kuleuven.ac.be

Abstract

During the last decades, under the increasing demand for more accurate high-performance motion systems, various friction models appropriate for control purpose of mechanical systems have been proposed. Most of these friction models formulate a heuristic dynamical model based on a few observed typical friction properties (e.g. a Stribeck curve for constant velocities and a hysteresis behaviour in presliding regime). This paper presents a novel friction model, called the generalized Maxwell-Slip (GMS) model, appropriate for control purpose, based on a physically motivated friction model, i.e. a generic friction model which simulates the contact physics at asperity level. This paper compares the GMS model with some well-known existing models and shows that the novel model is capable of predicting accurately not only the presliding regime and Stribeck effect, but also frictional lag, transition behavior, break-away force and the non-drifting (‘stiction’) property.

1 Introduction

Friction in mechanical systems is a nonlinear phenomenon which can cause control problems such as static errors, limit cycles and stick-slip. In order to design controllers for highly accurate machines, friction has to be taken into account. Friction is a result of extremely complex interactions between the surface and the near surface regions of the two interacting materials and other substances present such as lubricants and can thus not be simply predicted.

Detailed analysis of friction experiments reveals two friction regimes: the presliding regime and the sliding regime. In the presliding regime the adhesive forces (at asperity contacts) are dominant such that the friction force appears to be a function of displacement rather than velocity. This is so because the asperity junctions deform elasto-plastically thus behaving as nonlinear springs. As the displacement increases more and more junctions will break resulting eventually in gross sliding. In the sliding regime all the asperity junctions are broken such that the friction force is a function of the velocity (as described in classical friction models [2]). Accurate modelling and control of mechanical systems with friction requires a model which include both regimes.

Armstrong-Helouvry et al. [3] derive a general model structure which includes several experimentally observed friction properties. The model uses a switching function between the two regimes. Such a switching function is physically not justified and may result in implementation problems. In order to overcome this problem Canudas de Wit et al. [4] reformulated the model into a integrated friction model, known as the LuGre model, i.e. a set of dynamic equations integrating the sliding and presliding regime without the need of a switching function. Further extensions have been made to obtain more accurate models; e.g. the Leuven Model [13, 12] and the elasto-plastic model [6].

Most of these models are based on only one or two basic friction properties. The other friction behavior properties are more a coincidental consequence of the model formulation. The dissipativity of the models, which is an inherent property of friction, depends also on the parameters of the models or may sometimes be hard to prove. Therefore there is still a need in the control community to find a more suitable model. The authors proposed a physically motivated generic friction model [1]. This model is however computationally intensive, but it can be used (i) to extract a simplified version that is less computationally intensive than the generic model but more accurate than the existing models for control purposes and (ii) to compare with the existing models for control. The aim of this paper is to describe this derived model, which we shall call the generalized Maxwell-Slip (GMS) model, discuss its properties, and compare its behavior with the behavior of other existing friction models and the generic model.
Figure 1: Figure A shows the contact between two rough surfaces. Figure B shows the basic scenario for the generic model: the life cycle of one transformed asperity: (i) the original asperities make no contact, (ii) asperity interaction and (iii) lost of contact.

Section 2 describes briefly the physically motivated friction model. Section 3 discusses some existing friction models for control purposes, whereas in section 4 the GMS model is derived based on the generic model. In section 5 the different control friction models will be compared by applying different trajectories to the models.

2 The physical motivated friction model

The authors proposed a generic model at asperity level for friction force dynamics [1] and saw great similarities with the friction behavior measured by different authors and themselves [10]. The basic ingredients of this model fall into two categories: friction mechanisms and the asperity contact scenario. The first category comprises phenomenological mechanisms like normal creep of the contacting asperities, adhesion raising from a variety of sources and the hysteresis losses as a consequence of geometrical deformation of asperities. The second category of ingredients involves the asperity contact scenario, which deals with the transformation of two flexible surfaces to a set of flexible spring-mass elements, where each element has its own contact profile. Figure 1 shows the life-cycle of one such a transformed asperity contact. In case (i) there is no contact between the two asperities. In case (ii) there is contact and the corresponding deformation as a function of the displacement is represented by the lower surface. From the moment of contact a local friction coefficient, which increases with the contact time, is valid. When the asperities loose contact (iii) the deformation energy is lost.

This model is able to simulate all experimentally observed properties and facets of low-velocity friction force dynamics. This generic model has however the disadvantage that it is computationally intensive, at least 1000 equivalent asperities have to be modeled to obtain smooth signals, but it is useful for finding and validating simpler models which are appropriate for control: to the generic model arbitrary displacement trajectory can be applied which is not possible for a real setup due to the setup dynamics and friction force dynamics itself [10].

3 Extant friction models for control purposes

Dahl’s first paper [5] states: "The origin of friction is in quasi static bonds that are continuously formed and subsequently broken". This view on the origins of friction has been the starting point for several models like the bristle model [7], the Dahl model [5], the Karnopp-model [9] and many others.

The models discussed in this paper are limited to the most recent and effective ones: the LuGre model [4], the elasto-plastic friction model [6], the Leuven model [13] [12], and our novel model for control based on the generic model (see section 4).

3.1 The LuGre model

A model which is in line with Dahl’s considerations and employing also the idea of an averaged characteristic presliding displacement has been developed at the universities of Lund and Grenoble [4] and is called the LuGre-model. It combines the Dahl model with arbitrary steady-state characteristics such as the Strubeck curve $s(v)$. However, the interpretation of the internal state is that of the bristle model, i.e. friction is visualized as forces produced by bending bristles behaving like springs. Instead of modeling the random behavior of friction it is based on the average behavior of the bristles. The average deflection of the bristles is denoted as the state variable $z$ and modeled by:

$$\frac{dz}{dt} = v - \sigma_0 \frac{v}{s(v)} z,$$  \hspace{1cm} (1)

where $s(v)$, the Strubeck curve, is a decreasing function for increasing velocity bounded by an upper limit equal to the static force $F_s$ and a lower limit equal to the Coulomb force $F_c$. The friction force is given as a function of the state variable $z$ and the velocity $v$:

$$F_f = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 v.$$  \hspace{1cm} (2)

where the parameters $\sigma_0$, $\sigma_1$ and $\sigma_2$ equal the asperity stiffness, the micro-viscous friction coefficient and
the viscous friction coefficient. The LuGre model is a very popular model in the domain of control and simulation of friction due to its simplicity and the integration of presliding and sliding into one state equation. However, that is not to say that it models frictional behaviour faithfully.

3.2 The elasto-plastic model
According to Dupont et al. [6] the Karnopp, Dahl and LuGre model show drift behavior: a system, where the friction force is simulated by those models, subjected to an arbitrarily small bias and small vibrations, experiences drift. Dupont et al. use the term stiction for the phenomenon where the system does not experience drift. Due to the various definitions of stiction in literature, we prefer to use the term non-drifting property. The model introduced by Dupont et al., called the elasto-plastic friction model, is a general one state friction model that is stable, dissipative and which does not drift for presliding excitations. It is an extension of the LuGre friction model where the friction force equation remains the same (eq. 2) but the state equation changes to:

\[
\frac{dz}{dt} = v - \alpha(z, v)\sigma_0 \frac{v}{s(v)} z,
\]

and

\[
\alpha(z, v) = \begin{cases} 
0, & \text{if } |z| \leq z_{ba} \text{ or } sgn(v) \neq sgn(z) \\
1, & \text{if } |z| \geq z_{ss}(v) \\
\frac{1}{2} \sin \left( \pi \frac{s(v)}{z_{ss}(v) - z_{ba}} \right), & \text{else}
\end{cases}
\]

with \(0 < z_{ab} < z_{ss}(v) = \frac{s(v)}{\sigma_0}\).

The value \(z_{ba}\) indicates that for that friction state's range of motion, centered around the unloaded state, the friction is a result of a linear spring (with stiffness value equal to \(\sigma_0\)) and a linear damper force (with a viscous friction coefficient equal to \(\sigma_1 + \sigma_2\)). Within this range, no drift will occur. The disadvantage of this model is that it is still an approximation of the real behavior in presliding regime and the identification of the new parameter \(z_{ba}\) and the parameters \(\sigma_0, \sigma_1\) is not obvious. There is also no physical motivation for the choice of the particular shape of function \(\alpha(z, v)\).

3.3 The Leuven model
The Leuven model, presented by the authors [12, 13] was based on the experimental findings that the friction force in presliding regime is a hysteresis function of the position, with non-local memory, which was only approximated by the former models. The Leuven model tries to fit this specific behavior into the LuGre-model in order to obtain better tracking results at velocity reversals [11]. The equations of the Leuven model are:

\[
\frac{dz}{dt} = v \left( 1 - \text{sgn} \left( \frac{F_h(z)}{s(v)} \right) \left| \frac{F_h(z)}{s(v)} \right|^\delta_1 \right)
\]

\[
F_f = F_h(z) + \sigma_1 \frac{dz}{dt} + \sigma_2 v
\]

The hysteresis function \(F_h(z)\) can easily be implemented by using the Maxwell-Slip approximation [8].

4 The generalized Maxwell-Slip Friction Model

The former models are all empirical models, based on fitting a few typical basic friction properties in a dynamical model: (i) the friction force equals the Striebeck curve for constant steady state velocities (this property is incorporated in the LuGre, elasto-plastic and Leuven model), and (ii) the friction force is a function of position for the presliding regime. This property is incorporated in the Leuven model as hysteresis function of the position, with nonlocal memory. The LuGre and elasto-plastic model make only a rough approximation of the presliding behavior ignoring the nonlocal memory aspect. The other properties of the friction models, like the break-away force and frictional lag, are more a ‘coincidental’ result of the formulation of the model equations. Based on these two properties only, it is still possible to find numerous other models; e.g. another state equation for the Leuven model could be:

\[
\frac{dz}{dt} = v \text{ sgn} \left( 1 - \frac{F_h(z)}{s(v)} \right) \left| 1 - \frac{F_h(z)}{s(v)} \right|^\delta_1
\]

which still obeys the basic properties and is also capable to predict implicitly frictional lag and breakaway forces.

The reason models are fundamentally based on only two properties is the difficulty of measuring and imposing accurately the other phenomena. Based on the generic model briefly discussed in section 2, and which corresponds qualitatively with measured friction behavior [10], the challenge is now to extract a model applicable to control problems.

4.1 Methodology

The design of the novel model structure is based on the following three assumptions: (i) the LuGre equations (eq. 1 and 2) give an approximation of the sliding behavior but need to be adapted in order to describe the frictional lag phenomenon explicitly, (ii) the Maxwell-Slip model, used in the Leuven
model, may be used to model the behavior in presliding regime. It consists of several elementary friction blocks which are connected in parallel and (iii) in order to integrate both regimes into one model structure, the adapted LuGre equation is incorporated into each elementary friction block.

4.2 Sliding regime
In order to find the model equation in sliding regime the following assumptions are made: (i) the friction force \( F_f \) is proportional to the state variable \( z \):

\[
F_f = \sigma_0 z
\]  

with \( \sigma_0 \) the 'initial asperity stiffness'. This is based on the force equation of the LuGre model (eq. 2). The second term of eq. 2 was omitted due to the fact that it does not correspond to a physical explanation and the viscous term is not essential to our analysis being only a superposed term. (ii) The dynamic state equation of the deflection variable \( z \) is a general first order equation of the following form:

\[
\frac{dz}{dt} = v \left( 1 - \frac{z}{g^*(t)} \right) h^*(t)
\]

where the functions \( g^*(t) \) and \( h^*(t) \) have to be determined. For steady state behavior (\( \frac{dz}{dt} = 0 \)) with a constant velocity the deflection variable \( z_{ss} \) equals \( g(t) \) and the friction force \( F_{f,ss} \) equals the Stiebeck value \( s(v) \). Using this property the initial assumptions can be rewritten as:

\[
\frac{dF_f}{dt} = v \left( 1 - sgn(v) \frac{F_f}{s(v)} \right) h(t)
\]

where the state variable \( z \) and the initial stiffness \( \sigma_0 \) are eliminated and the unknown function \( h(t) \) corresponds to the unknown function \( \frac{h^*(t)}{\sigma_0} \).

To determine the unknown function \( h(t) \) the frictional memory in sliding regime can be used. In order to examine this, a unidirectional displacement signal has to be applied. The chosen velocity signal is shown in figure 2A. The resulting friction force \( F_f \) and Stibbeck values \( s(v) \), based on the generic model, are shown in figure 2B. Figure 2C shows the frictional lag in the sliding regime; i.e. the friction force as a function of the velocity. Extracting \( h(t) \) from eq. 7 yields:

\[
h(t) = \frac{dF_f / dt}{v \left( 1 - \frac{F_f}{s(v)} \right)}
\]

where \( h(t) \) is expressed as a function of signals from the generic model. The dotted line of figure 2D shows function \( h(t) \) calculated using equation 8. The result is not a constant equal to one, like in the LuGre model, but can be approximated by a function of the form \( \frac{h}{|h|} \) (full line in figure 2D) . Consequently the proposed model in sliding is of the following form:

\[
\frac{dF_f}{dt} = sgn(v)C \left( 1 - \frac{F_f}{s(v)} \right)
\]

The main difference between the LuGre or the Leuven state equations and equation 9 is the absence of the velocity as a separate factor in the state equation.

4.3 The presliding regime
The presliding regime has not been considered yet. As shown in the modifications of the Leuven model [12], the presliding regime can be represented by a Maxwell-Slip model (see figure 3). This model treats the hysteresis as a parallel connection of \( N \) elementary friction models. Each elementary friction model corresponds to an asperity, where each asperity can stick or slip. If asperity \( i \) sticks, the elementary friction force \( F_i \) is proportional to the deflection of the asperity, which can be modeled as:

\[
\frac{dF_i}{dt} = k_i v,
\]
with \( k_i \) the stiffness of the asperity. The asperity will slip if the elementary friction force equals the maximum value \( W_i \) that it can sustain. From then on the element begins to slip and the force equals to the maximum force, and by the classical Maxwell-Slip model:

\[
\frac{dF_i}{dt} = 0.
\]

Using the Maxwell-Slip model, where the total friction force \( F_j = \sum F_i \) equals the sum of all the asperity forces, it is possible to model a hysteresis function with nonlocal memory. The number of possible memory locations depends on the number of elementary friction models used and the shape of the hysteresis curve depends on the chosen parameter sets \( k_i \) and \( W_i \). We need now only to incorporate the transition to sliding; i.e. an appropriate equation to replace equation 10.

### 4.4 Merging of the presliding and sliding equations

One way to merge the adapted equation into the Maxwell-Slip model is by splitting up equation 9 into several elementary sliding equations for each elementary model:

\[
\frac{dF_i}{dt} = \text{sgn}(v)C \left( \alpha_i - \frac{F_i}{s(v)} \right),
\]

which replaces equation 10. For constant velocity sliding each elementary model has a friction force equal to \( W_i(v) = \alpha_i s(v) \). If \( \sum \alpha_i = 1 \) the total friction force for constant velocities equals the Stribeck curve. If all the asperities are slipping, the global friction behavior will be the same as in equation 9. There are other possibilities to split equation 9 up into several elementary equations, but the chosen one has a minimum set of parameters which have to be estimated.

### 4.5 Resulting novel friction model

The generalized Maxwell-Slip (GMS) friction model is based explicitly on three friction properties: (i) the Stribeck curve for constant velocities, (ii) the hysteresis function with non-local memory in the presliding regime, and (iii) the frictional memory in the sliding regime. The developed model is in fact a parallel connection of different single state friction models, all having the same input namely the velocity. The friction force is given as the summation of the outputs of the \( N \) elementary state models plus an extra viscous term, if viscous friction is present at the interface. (In this way, viscous friction is lumped in one parameter for the whole system. In reality the viscous effect may have its own dynamics per elementary part):

\[
F_j(t) = \sum_{i=1}^{N} F_i(t) + \sigma_2 v(t)
\]

The dynamic behavior of each elementary model can be written as:

- If the elementary model is sticking, the differential equation is given by:

\[
\frac{dF_i}{dt} = k_i v,
\]

the elementary model remains sticking until \( F_i > \alpha_i s(v) \).

- If the elementary model is slipping, the differential equation is given by:

\[
\frac{dF_i}{dt} = \text{sgn}(v)C \left( \alpha_i - \frac{F_i}{s(v)} \right).
\]

The elementary model remains slipping until the velocity goes through zero.

### 4.6 Properties of the model

The friction force in sliding regime will always be bounded by the static friction \( F_s \) and Coulomb friction \( F_c \).

The differential equation (equation 9) can be written as a first order equation:

\[
\frac{|s(v)|}{C} \frac{dF_j}{dt} + F_j = s(v),
\]

with as input the Stribeck curve \( s(v) \) and a time constant equal to \( \frac{C}{|s(v)|} \). This means that the friction force in sliding regime will always be attracted to the Stribeck curve. Given the fact that \( F_c < |s(v)| < F_s \) and that the initial sliding friction force equals the Stribeck value at initial sliding velocity, it is easy to prove that the friction force during sliding will lie between \( F_c \) and \( F_s \); i.e. \( F_c < |F_j| < F_s \). Therefore the absolute value of the friction force during sliding and presliding has an upper bound equal to the static force and a lower bound equal to the Coulomb force.

**The friction force is a continuous function.**

This is a trivial property because the output of the elementary model is the derivative of the elementary friction force which is bounded, due to the bounded velocity signal (for presliding regime) and the bounded friction force signal and nonzero \( s(v) \) value (for the sliding regime).

**Dissipativity of the model**

The new model will be dissipative if all the elementary friction blocks are dissipative. In the case of
Table 1: Parameters used for the comparison of the different models. The parameters are estimated out of the generic model results. The parameters $F_m$, $k$, and $\alpha$ can be estimated by curve fitting of a hysteresis curve of the generic model.

<table>
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<tr>
<td>$N$</td>
<td>24</td>
</tr>
</tbody>
</table>

5 Comparison of the different friction models for control purposes

This section compares the different mentioned friction models. The parameters shown in table 1 are chosen such that the behavior of the mentioned friction models can be compared with the behavior of the generic model. Using these parameters the presliding behavior, the frictional lag, and the transition behavior are compared.

5.1 Steady state velocity

Whereas the Dahl model and the Bliman model always give the Coulomb friction $F_c$ for different steady state velocities, the other models are capable of describing the steady state friction characteristic. For the LuGre, the elasto-plastic, the Leuven and the GMS model the friction force at steady state velocity equals:

$$F_{f,ss}(v) = s(v) + \sigma_2 v.$$

5.2 Presliding regime

In order to investigate the hysteretic behavior of the friction force in presliding regime the applied position signal is chosen such that the existence of non-local memory in presliding regime is visible. Figure 4 shows the basic position signal as a function of time. To prove the velocity independence in the presliding regime the same position signal is applied 5 and 20 times faster than the basic one shown in figure 4. Figure 5 shows the resulting friction forces as a function of the position for different models and different velocities. These figures show clearly that the nonlocal memory of the hysteretic function cannot be represented by the LuGre model. Neither is the Dahl model capable to represent it, but it has an extra shape factor to better approximate the overall transition curve of the hysteresis. The results of the Leuven and GMS model corresponds fairly well to the generic model results and both are capable of incorporating the nonlocal memory; i.e. when an inner loop is closed the force follows further the former outer loop. In order to obtain a hysteretic behavior which is time-independent, the Leuven model requires that $\frac{dz}{dt} = v$ yielding $F_f$ equal to $F_h(x)$. This can be accomplished by a high shape factor $\delta_l$ and a small micro viscous friction coefficient $\sigma_1$. For larger micro viscous coefficients the hysteresis of the LuGre and Leuven model becomes rate-depended.

5.3 Frictional lag

To investigate the frictional timelag behavior of the different models a unidirectional velocity signal is applied, i.e. a sine plus an offset signal. Figure 6 shows the frictional lag for the generic, LuGre, Leuven, and GMS model. The LuGre and Leuven model on the one hand and the generic and GMS model on the
other hand have similar shapes. This is due to the fact that the state equations of the LuGre and Leuven model are proportional to the velocity which is not the case for the GMS model. All the models show a lower friction force for decreasing velocities than for increasing velocities and they all show an increasing enclosed surface for increasing frequency.

5.4 Transition behavior
The difference between the discussed models is also clearly visible if a pure sine signal is applied as velocity signal. Such a signal enables us to look at the transition from presliding regime to sliding and vice versa. Figure 7 shows the friction force as a function of the velocity for the generic, LuGre, Leuven and the GMS model. The LuGre and Leuven model on the one hand and the generic model and the GMS model on the other hand show the same behavior. This is again the result of the different state equations. Another remarkable point is that for the last two models the maximum force at increasing positive velocity is higher than the maximum force at decreasing positive velocity, experiments confirm this behaviour, which is not the case for the other two models. For different frequencies of the sine velocity signal the generic model and the GMS model still have a similar behavior.

5.5 Non-drifting properties
Dupont et al. proposed the following test to check this property: apply an increasing force to a mass until the mass starts to slide. After sliding apply a constant force and subsequently a sine signal superimposed on that constant force for which in both cases the total applied force is less than the static friction. They state that the mass will not drift away during the sine excitation: it sticks within a certain presliding region.

The applied force shown in figure 8 was chosen to validate the stiction capability of the models, the force ramps up to cause break-away, with a resulting break-away force, and then returns to a level below the Coulomb friction. An additional oscillation is present, such as could be introduced by sensor noise or vibration. Figure 9 shows the friction force as a function of the position. The LuGre model exhibits presliding displacement and there is a steady state drift in position during the oscillating period. The elasto-plastic model exhibit presliding displacement and has a limited drift. The drift stops from the moment $|z| < z_0$. The Leuven and GMS model have the same behavior: they exhibit presliding displacement and no drift occurs as soon as the friction force becomes smaller than the break-away force, which corresponds to the measured results [10].

5.6 Breakaway force
When the driving force is ramped up with a constant rate, the friction force opposing the driving force also increases as long as the system sticks. When the system breaks away and starts to slide, the friction force reaches a maximum force, called the break-away force. Figure 9 shows also that the LuGre model, elasto-plastic model, Leuven model, and the GMS model are all capable of simulating the break-away force.
away friction behavior. In correspondence with experimentally measurements, the breakaway force decreases with increasing rate of applied force for all the discussed friction models.

6 Conclusions

This paper briefly discusses some recent friction models for control purposes. Based on a physically motivated friction model, which is computationally too intensive to use for control, a novel friction model, the Generalized Maxwell-Sip (GMS) friction model, is derived. The model consists of a parallel connection of different elementary friction models, each with the same structure but with a different set of parameters. This model has the same basic properties as the existing friction models (hysteresis behavior in presliding and a stribeck curve for constant velocities), but the property of frictional lag is taken explicitly into account in the new model. This results in a better correspondence with the results of the physically motivated friction model in the case of frictional lag and transitional behavior, without adding extra parameters in the model compared to the existing models. The other frictional behaviour types as break-away force, non-drifting property and stick-slip phenomena can also be modeled by the GMS model.

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