Constrained estimation with truncated Gaussians

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1 Introduction

The state estimation problem consists of estimating the hidden state of a dynamic system, governed by a stochastic linear or nonlinear process model, from a set of noisy observations, which depend on the hidden state by a stochastic linear or nonlinear observation model. This problem is often solved recursively using Bayesian filtering. In the recursive Bayesian filtering approach, a new state estimate is determined at every time-step, given the latest measurement and some knowledge of the state at the previous time-step. However, incorporating a priori knowledge in the form of equality or inequality constraints on the states proves to be difficult and is still a topic of ongoing research [1, 2].

Another popular approach to on-line state estimation, is moving horizon estimation (MHE) [3], which can be considered to be a modified batch method. An important advantage of MHE over the Gaussian filter variants [6] is that it is quite natural to take into account a priori knowledge in the form of inequality constraints [4]. However, in the general nonlinear case, the MHE approach requires solving a constrained nonlinear program at every time-step, which may prove to be difficult and computationally expensive.

2 Method

Instead of turning to MHE for constrained estimation, a novel idea is introduced based on modified recursive Gaussian filters for constrained estimation by considering the class of truncated Gaussians [2]. By considering this class of distributions, prior knowledge or considerations based on physical insight can be taken into account in the form of inequality constraints on the states. Specifically, we discuss the class of Gaussians truncated outside hyperrectangles. A constrained recursive estimation algorithm is developed, based on modified Gauss-Hermite quadrature rules [6] to evaluate integrals numerically over bounded hyperrectangles. The methodology is not limited to linear systems and is easily applicable to one-dimensional systems, but extension to more-dimensional problems poses some problems. An approximate extension for more-dimensional systems is presented. Simulation results are presented which indicate the benefit of the proposed method both in the one-dimensional and more-dimensional case.

3 Conclusions and future work

It is shown that the class of truncated Gaussians proves to be very useful for constrained estimation. Applications include constrained estimation for use in closed-loop, a better calculation of the arrival cost in MHE and the possibility to include inequality constraints based on physical insight or a priori knowledge. Future work includes extension to domains more general than hyperrectangles.

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